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APPLICATION OF A 2D MARKOVIAN APPROACH TO THE MODELLING OF SOUND PROPAGATION IN STREETS

Zaiton Haron[1,*], David Oldham[2] , Khairulzan Yahya[1] , Rozana Zakaria[1]

[1]*Construction Technology and Management Centre, Faculty of Civil Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor Bahru, Malaysia*
[2]*Acoustic Research Unit, University of Liverpool, UK*

*****Corresponding Author: zaitonharon@utm.my

Abstract: Markov approach has been used in modeling multiple reflections or reverberation in rooms. This paper examines the application of Markov approach in the study of propagation of sound in streets by treating multiple reflections in streets as a Markov process. In this preliminary study a two dimensional model has been employed. Results obtained are compared with those obtained using ray tracing, RAYNOISE. It is concluded that the sound field predicted by the Markov process is similar to the sound field obtained by the ray tracing model using the diffuse option and it is suggested that the method can be extended to account for application in 3D streets.

Keywords: *Diffuse reflection, Markov process, Transition probability, Sound propagation*

1.0 Introduction

Propagation characteristic of city streets is currently a topic of considerable interest. A number of models have been developed based upon façade reflections (Ismail and Oldham, 2003). Models may be either specular (mirror-like reflection) (Wiener et al., 1965; Radwan and Oldham, 1987; Diggory and Oakes, 1980 and Oldham and Radwan, 1994), diffuse (reflected at a number of angles-the complement to specular reflection) (Kang, 2002a-b; Kang 2002; Kang 2005 and Picaut et al., 2002) or mixed between specular and diffuse (Bullen and Fricke, 1977; Davies, 1978 and Wu and Kittinger, 1995). Most software employ mixed model. However Ismail and Oldham (2003) found that for the case of mixed specular and diffuse reflections, the specular component will diminish rapidly with increased orders of reflection leading to the dominance of diffuse sound fields and thus the diffuse models will tend to be more accurate.

In recent years extensive research have been carried out to study the usage of diffuse facade reflections as the basis for modelling of noise propagation in urban streets. Kang (2000) extensively employed the diffuse reflection through radiosity approach and established the suitability of the techniques in reducing noise in streets. The technique which was initially used in lighting and extended for room acoustic has been found suitable for carrying out parametric investigation. Kang found good agreement with experimental measurements. Picaut et al. (2002) studied with a different approach, i.e classic diffusion equation which requires the determination of a diffusion coefficient for the sound particles.

Several researches have focused on the use of Markovian approach in treating the sound propagation in diffuse field. Gerlach (1975) studied the Markov approach to enable a systematic investigation of the effect of the distribution of surface treatments on the reverberation time of a room. Gerlach proposed that reverberation in a room with diffusing surface could be modelled as a Markov process. Gerlach studied that the energy falling on a surface of an enclosure will be reflected and distributed to all other surfaces in a room according to their “visibility” with respect to the reflecting surface as Markov Chain. This technique has been extended by Kruzin and Fricke (1982) to the study of sound propagation in an enclosure with obstructions. More recently Alarcão and Bento Coelho (2003) described the application of Markovian techniques for the study of auditorium acoustic. However, studies involving the use of the method in modeling of noise propagation in street have not yet been reported.

This paper examined the preliminary application of Markovian approach in modeling of noise propagation in streets with the street modeled as two dimensional or 2D streets. The results of the Markov model for 2D streets simulations were compared with the results obtained from the RAYNOISE model using the diffuse reflection option.

2.0 Theoretical Consideration

The model is developed base on the assumption originally made by Gerlach (1975) and Kruzin and Fricke (1982) for the case of a room, that multiple reflections in a street with diffusely reflecting surfaces can be modeled as a Markov process. The model assumes the facades to have irregular surfaces and thus to reflect sound diffusively. The source and receiver is shown in model configuration in Figure 1. The model divides all wall surfaces up to a number of small patches, i.e. i=1,2…n, and j=1,2….n for surface I and surface J, respectively.

The model assumes that the source initially radiates the sound energy cylindrically to the patches which can then be regarded as secondary sound sources. The basic principle of the source energy distribution is that the fraction of energy incident on each patch is equal to the ratio of the angle subtended by the receiving patch divided by the total angle into which energy from the source radiates. The normal intensity at the centre of a patch can be determined using the inverse distance law which applies to a line source.

$$e_i = \Delta_x W_a \cos\theta_i \frac{1}{2\pi d_{si}} \tag{1}$$

Where W_a is the source of sound power per unit length, Δ_x is the element width and d_{si} is the distance from the centre of patch i to the source and $\cos\theta_i$ is the angle of incidence of a sound ray from the source to the centre of the patch.

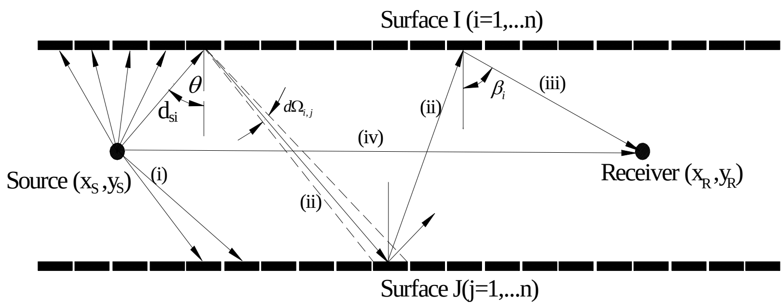


Figure 1: Distribution of sound energy from source to patches, (i) patches to patches, (ii) patches to receiver, and (iii) direct sound from source to receiver (iv) in 2D Street

The initial energy on each patch on surfaces I, E_I^{0} and J, E_J^{0} are accumulated in sub-vectors as follows;

$$\begin{aligned} E_I^{(0)} &= (e_1^{(0)}, \ldots, e_n^{(0)}) \\ E_J^{(0)} &= (e_1^{(0)}, \ldots, e_n^{(0)}) \end{aligned} \tag{2}$$

The initial energy on surfaces I and J can be described in terms of vector E^0 which can be written as;

$$E^{(0)} = \left\lfloor E_I^{(0)} \quad E_J^{(0)} \right\rfloor \tag{3}$$

E^0 will be redistributed to other surfaces during the first transition. Transitions correspond to orders of reflection. The energy will be reduced by the absorption of each patch, for example, a fraction $(1-\alpha_i)$ of the initial energy on patch i is distributed to patch j according to the transition probability and a fraction $(1-\alpha_j)$ to patch i. Thus the total energy distributed from patch i to patch j at the first transition is given by ;

$$E_J^{(1)} = \sum_{i=1}^{n} e_i^{(0)} (1-\alpha_i) p_{ij} ; (j = 1....,n) \tag{4}$$

The energy distribution from both surfaces at the first transition now becomes;

$$E^{(1)} = E^{(0)} (P_e) \tag{5}$$

where P_e is effective transition matrix.
$E^{(1)}$ will again be transmitted to other surfaces in second transition and after the q-th transition the energy distribution $E^{(q)}$ is given by;

$$E^{(q)} = E^{(q-1)} P_e = E^{(0)} (P_e)^q \tag{6}$$

At each transition order energy distribution occurs between patches and the patches will also reflect sound energy diffusely to a receiver (Figure 1). For example, the intensity from patch i to the receiver obtained from the inverse distance law and assuming that Lambert's cosine law applies, is given by;

$$I_i = \frac{e_i \cos \beta_i (1-\alpha)}{\pi d_{ir}} \tag{7}$$

where d_{ir} is the distance from the centre of the i-th patch to the receiver and β_i is the angle to normal patch of a ray from the centre of the patch to the receiver and α is the absorption coefficient of the patch.

The source also distributes sound energy directly to the receiver. The intensity at the receiver can be determined using the inverse distance law which applies to a line source;

$$I_d = \frac{W_a}{2 \pi d_{sr}} \tag{8}$$

where I_d is the intensity of direct sound, W_a is the sound power per unit length of the source, and d_{sr} is distance from the source to receiver.

The energy response at the receiver can be determined by taking into account all orders of transition from patches to receiver. For q orders of transition the energy at receiver R can be written as:

$$I_T = I_d + \sum_{j=1}^{q} E^{(q-1)} AR \tag{9}$$

The sound pressure level SPL at receiver is calculated using;

$$SPL = 10 \lg 10 \left(I_T / 10^{-12} \right) \tag{10}$$

3.0 Modeling Algorithm

3.1 Transition Probability Matrix

The key in modelling is derivation of transition probabilities between patches in sub-matrices which relate to the sound radiation between patches on pairs of walls.

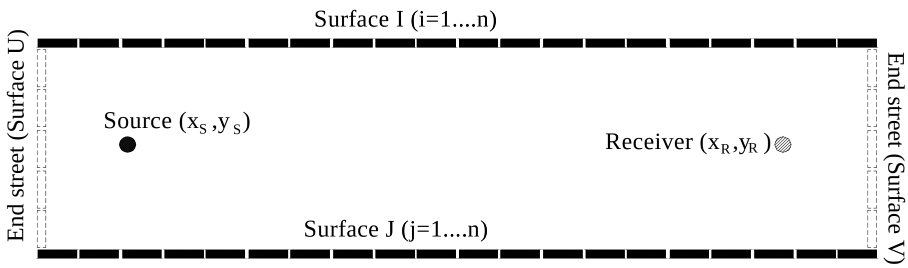


Figure 2: 2D model configuration (2D room becomes 2D street)

In 2D room, the transition matrix P consist of 4x4 sub-matrices each relating to the sound radiation between patches on pairs of walls. These sub-matrices can be arranged in the form of the effective transition matrix P as shown in Equation 11.

$$P_e = \begin{bmatrix} 0 & (1-\alpha_I).P_{IJ} & (1-\alpha_I).P_{IU} & (1-\alpha_I).P_{IV} \\ (1-\alpha_J).P_{JI} & 0 & (1-\alpha_J).P_{JU} & (1-\alpha_J).P_{JV} \\ (1-\alpha_U)P_{UI} & (1-\alpha_U).P_{UJ} & 0 & (1-\alpha_U).P_{UV} \\ (1-\alpha_V).P_{VI} & (1-\alpha_V).P_{VJ} & (1-\alpha_V).P_{VU} & 0 \end{bmatrix} \tag{11}$$

P_{IJ} represents the sub-matrix for transition probability between the patches on wall I and J and accordingly to other sub-matrices. Nulls represent the sub-matrix for transition probabilities between the same walls such as surface I to I which do not exist. $(1-\alpha_I)$ represents the fraction of sound energy incident on surface I which radiates to surface J where α_I is the reflection coefficient.

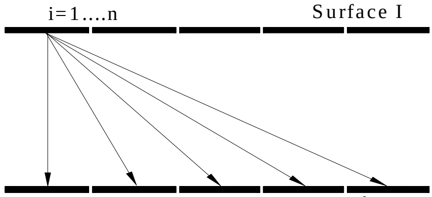
The 2D street transition probabilities is derived using Equation 11 where there will be no sound reflection from the ends of street (wall U and V) thus the reflection coefficient of the street ends is equal to zero. Also by considering response at receiver the effective transition matrix P_e becomes:

$$P_e = \begin{bmatrix} 0 & P_{IJ_e} \\ P_{IJ_e} & 0 \end{bmatrix} \tag{12}$$

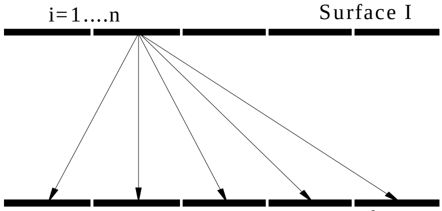
Sub-matrix P_{IJ} consists of transition probability matrix between the patches on surfaces I and J which is denoted by $p_{i,j}$, thus;

$$P_{IJ} = \begin{bmatrix} p_{1,1} & . & . & p_{1,n} \\ . & . & . & . \\ . & . & . & . \\ p_{n,1} & . & . & p_{n,n} \end{bmatrix} \tag{13}$$

The transition probabilities from patches j to patches i will be identical. Figure 3 shows the radiation of sound from the centre of the first and second patches on surface I to the patches on surfaces J.



(a) From a patch on surface I to patches on surface J



(b) From the next patch on surface I to patches on surface J

Figure 3: Radiation of sound between patches

$p_{i,j}$ are obtained using the following;

$$p_{i,j} = d\Omega_{i,j} / \pi \tag{14}$$

$d\Omega_{i,j}$ is the angle subtended by patch j at the centre of patch i when the patches radiate sound cylindrically into π radians.

What makes the Markov method attractive is that there will be a pattern by which these transition probabilities will be repeated. For example, in Figure 3 and Table 1 it can be seen that $P_{i,k} = P_{i+1,k+1}$ and also $P_{i,k+1} = P_{i+1,k+2}$.

Table 1: Matrix of transition probabilities for five elements per side.

To	From									
	Top surface					Bottom surface				
	i=1	i=2	i=3	i=4	i=5	k=1	k=2	k=3	k=4	k=5
i=1	0	0	0	0	0	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	$P_{1,5}$
i=2	0	0	0	0	0	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$
i=3	0	0	0	0	0	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$
i=4	0	0	0	0	0	$P_{1,4}$	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$
k=5	0	0	0	0	0	$P_{1,5}$	$P_{1,4}$	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$
k=1	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	$P_{1,5}$	0	0	0	0	0
k=2	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	$P_{1,4}$	0	0	0	0	0
k=3	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$	$P_{1,3}$	0	0	0	0	0
k=4	$P_{1,4}$	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$	$P_{1,2}$	0	0	0	0	0
k=5	$P_{1,5}$	$P_{1,4}$	$P_{1,3}$	$P_{1,2}$	$P_{1,1}$	0	0	0	0	0

3.2 Distribution of sound from the source to patches

The source power and patch width can be separated from Equation 1 to obtain a source distribution function. For example source distribution function to wall I, s_i can be described as;

$$s_i = \frac{\cos\ \theta_i}{2\pi d_{si}} \tag{15}$$

For surfaces I and J there will be n source functions and can be written as;

$$S = \begin{bmatrix} S_I & S_J \end{bmatrix} \tag{16}$$

3.3 Distribution of Sound from Patches to a Receiver

The energy and absorption terms can be separated from Equation 7 to yield a receiver function. For patch i the function can be written as;

$$r_i = \frac{\cos \beta_i}{\pi d_{ir}} \tag{17}$$

where d_{jr}, the distance from the centre of the j-th to receiver and β_j angle normal to patch of a ray from the centre of the patch j.

The receiver functions from surface I, J, can be arranged in sub-matrices R_I , R_J respectively. For surfaces I and J there are 2n patches and they can be written in the form of a vector array as follows;

$$R = \begin{bmatrix} R_I & R_J & \end{bmatrix} \tag{18}$$

If the receiver is moved along the road by a distance equal to one patch width then a new vector can be easily constructed from the established distribution of functions.

4.0 Results and Discussion

A computation was carried out for a street with length L=100m, and width w=10m. The geometry is illustrated in Figure 4. A cylindrical source was located at (5m, 5m) with a power of 0.002watts/m and the receivers were positioned at intervals of 5m along a line mid-way between the facades from 10-95m. A length of 100m was chosen based on the assumption that in urban areas propagation effects are important for distance up to 200m (Oldham and Radwan, 1994). The width of 10m was chosen as a width of less than 10m would mean that the street is classified as narrow and propagation might be affected by interference effects due to multiple reflections from buildings and the ground (Iu and Li, 2002). The model walls were assumed to have a uniform absorption coefficient of 0.1. This is based on the assumption that the façades consist of large areas of brickwork (Delany, 1972). Lee and Davies (1975), Oldham and Radwan (1994) and Kang (Kang, 2000a-b; Kang 2002a-b and Kang, 2005) also used an absorption coefficient 0.1 in their models.

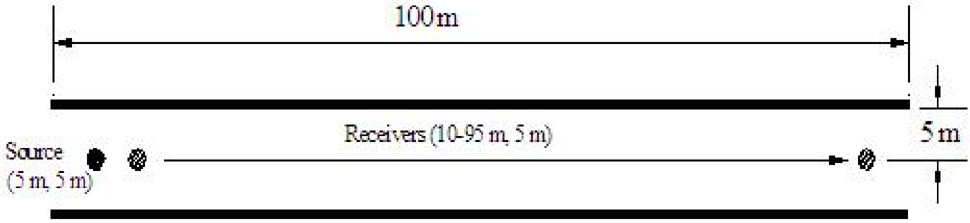


Figure 4: A 2D Street configuration

The calculation of the cumulative energy from surfaces I and J at a receiver positioned 10 m from the source is shown in Figure 5. The sound energy reflected from the patches reached a steady state value after approximately 12 transition orders. The energy response at the receiver is the cumulative sum of energy obtained for 12 transition orders and the direct sound energy. By taking into consideration all receivers along the street from 10-90 m at intervals of 5m, the sound pressure level was calculated and plotted as shown in Figure 6. It can be seen that the sound pressure level variation/doubling distance becomes higher when the source-receiver distance is greater.

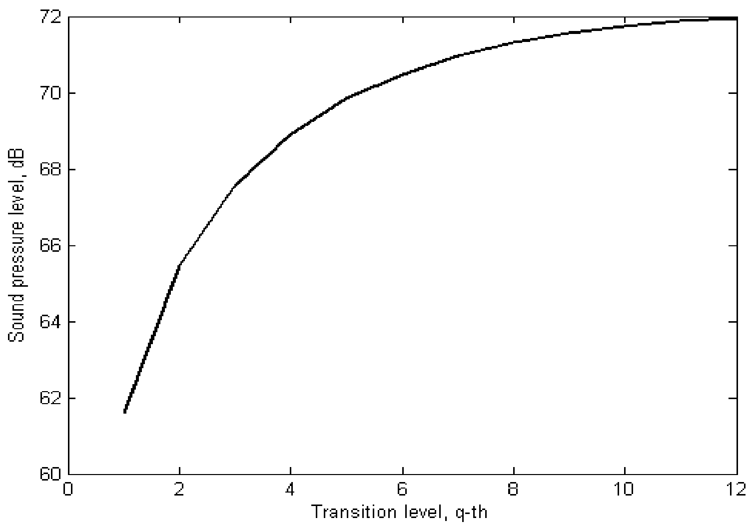


Figure 5: Cumulative sound pressure level after each transition order

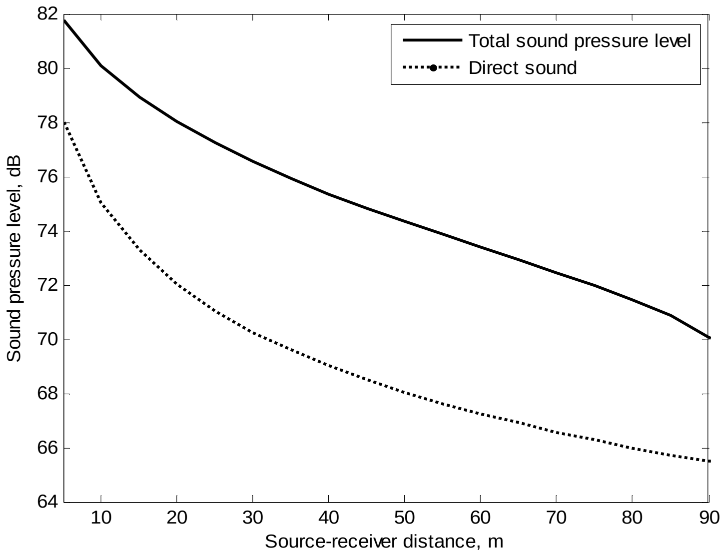
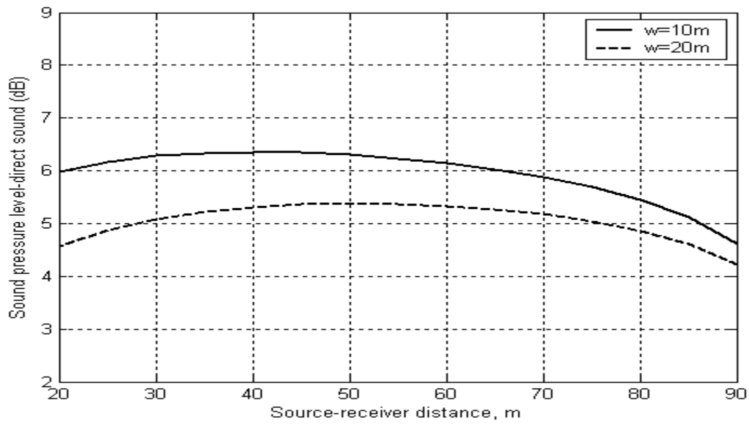


Figure 6: Sound pressure level along the street length for source-receiver distance 5 to 90m

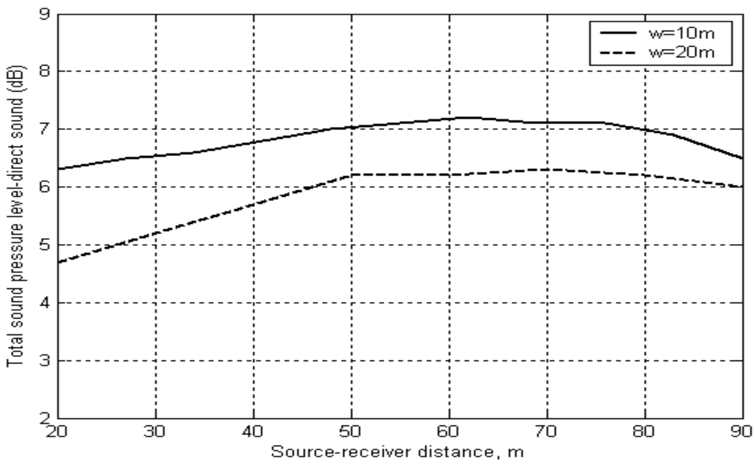
5.0 Comparison of the Results with Raynoise Model

RAYNOISE uses hybrid between mirror image source methods and Monte Carlo method to model the diffuse reflection. The software requires beam tracing using cone or triangular beams to determine the position of source images. A RAYNOISE model which satisfied the condition in Figure 4 was employed. The street width of 10m and 20m were utilized. The model was run with 300,000 rays with the triangular beam option, 30 reflections and the diffusion coefficient, d, equal to 1 to obtain totally diffuse reflections. The difference in relative sound pressure level along the street for each width for the two methods was about 2 dB at a source-receiver distance 90m (Figure 7).

Similar trends were found with the noise level at any point along the street being greater for the narrower street. Markov method tend to show lower noise levels than those for the RAYNOISE method however, given the use of a two dimensional model for the Markov technique, some discrepancy is to be expected. The reduction of sound attenuation near the source becomes significantly different when the width is increased to 20m. This is due to the angle subtended by a patch becoming smaller as the width increases and as a result the transition probability or probability of sound radiating from one patch to another becomes smaller. As the angle subtended becomes smaller this also means that more energy will be radiated out of the street ends. The same trend was observed for the RAYNOISE model. The fact that a wider street has a lower sound pressure along the street length has been reported by Oldham and Radwan (1994) and Kang (2002).



(a) Markov Method



(b) RAYNOISE Model

Figure 7: Comparison of street propagation characteristics

6.0 Conclusion

The use of Markov chain in the study of the propagation of sound in streets has been proposed. In this preliminary study a 2dimensional model has been employed. The relative sound pressure level prediction by the 2D Markov process showed agreement (within 2 dB) with the results obtained from a RAYNOISE model. Agreement within 1 dB-2dB was also observed with the prediction of the effect of street width. The results of this initial study suggest that the sound field predicted by the Markov process is similar to the sound field obtained by a ray tracing model using the diffuse option. Further work is required to extend the work in order to account for the effect of the height of the street facade to capture the situation in real 3D empty streets.

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