Malaysian Journal of Civil Engineering 27(2):290-304 (2015)

INTERRELATION AMONG PERTINENT ELEMENTS OF CRITICAL THINKING AND MATHEMATICAL THINKING IN THE REAL-WORLD PRACTICE OF CIVIL ENGINEERING

Sharifah Osman[1]*, Salleh Abu[2], Shahrin Mohammad[3] & Mahani Mokhtar[2]

[1]*Centre for Engineering Education, Universiti Teknologi Malaysia, Skudai, Malaysia*
[2]*Faculty of Education, Universiti Teknologi Malaysia, Skudai, Malaysia*
[3]*Faculty of Civil Engineering, Universiti Teknologi Malaysia, Skudai, Malaysia*

Corresponding Author: sharifah8283@gmail.com*

Abstract: Engaging critical thinking and mathematical thinking in solving engineering problems is complying ABET engineering criteria. Thus, the understanding of interaction between these two thinking is critical for the current engineering education. However, the interaction is not thoroughly being studied in the real-world engineering practice. This paper describes the selection process of the pertinent elements of critical thinking and mathematical thinking and the interrelation among the elements in the real-world civil engineering practice, using modified grounded theory analysis. Data consisted of semi-structured interviews with eight practicing civil engineers from two consultancy firms. A total of fifty three pertinent elements emerged during open coding process. Axial coding process developed the interrelation among the pertinent elements. The findings showed that during design process, the elements were interwoven, concurrently used, indispensable and inexorably linked. Thus, the results provide main source of information to explain the interaction among pertinent elements in selective coding process.

Keywords: *Critical thinking, mathematical thinking, modified grounded theory, qualitative research, research methodology, engineering education*

1.0 Introduction

Critical thinking has been one of the highly valued emphases of students' outcomes today not only in academic settings of higher education but as well in professional environments (Facione, 1990; Paul, 1995). It is called "critical' not because it is negative or accusatory, but because it judges according to prescribed criteria (Beyer, 1990).

Kadir (2007) states there are many different definitions of critical thinking from different perspectives. However, there is still no universal consensus on a definition of critical thinking amongst educators, philosophers and psychologists in the field. Thus despite the growing body of literature on critical thinking, consensus on a definition remains elusive. The lack of unity in defining critical thinking has contributed to the varied definitions of critical thinking abilities and dispositions amongst educators, philosophers and psychologists in the field (Kadir, 2007). The national panel of experts in the Delphi Project (Facione, 1990, 2007) was eventually reached to a conclusion that critical thinking encompasses two important dimensions; the cognitive skills dimension and the dispositional dimension. The abilities consist of six elements of cognitive skill such as interpretation, analysis, evaluation, inference, explanation and self-reflection. While, dispositions comprising systematic (orderliness), inquisitiveness, judicious (maturity), truth seeking, confidence, open mindedness, and analyticity (Facione, Facione *et al.*, 2000; Facione, 2013)

Besides, in the twenty-first century, everyone can benefit from being able to think mathematically because it is valuable as a powerful way of thinking about things in the world (Devlin, 2002). Mathematical thinking is important in a larger measure as it equips ones with the ability to use mathematics (Stacey, 2007). This is not the same as doing mathematics which usually involves the application of formulas, procedures, and symbolic manipulation. Devlin (2012) argues that mathematical thinking does not have to be about mathematics at all, but parts of mathematics provide the ideal target domain to learn how to think logically, analytically, quantitatively, and with precision. In addition, Schoenfeld (1992) also mentions that mathematical thinking is not merely involved mathematical content knowledge. The ability to think mathematically and to use mathematical thinking to solve problems is an important goal of schooling in such a way that mathematical thinking will support science, technology, economic life and development in an economy (Stacey, 2007).

Similarly, there are different definitions and interpretations of the term mathematical thinking in literature based on experts view as well as scholars' definitions. Yet, there is no consensus on what mathematical thinking is (Sternberg, 2012). However, a lot of accordance in saying that the mathematical thinking is not a natural way of thinking; it needs to be taught and can be learnt (Devlin, 2012; Katagiri, 2004; Stacey, 2007). Schoenfeld (1985, 1992) describes mathematical thinking as the ability to implement five aspects of cognition namely the knowledge base, problem solving strategies or heuristics, monitoring and control, beliefs and affects and practices.

On top of that, findings from the previous studies have shown congruence between critical thinking and mathematical thinking in the real civil engineering workplace context (Radzi *et al.*, 2011 & 2012). However, there is a lack of literature which indicates comprehensive overview, and research that rigorously investigates interaction between critical thinking and mathematical thinking in the real-world engineering practice. In addition, there is no theory pertaining to the understanding of the process which may relate the mathematical thinking to the critical thinking. Therefore, to have an empirical insight into the interaction among pertinent elements of critical thinking and mathematical thinking becomes the main goal of this study.

This study adopts qualitative research method with modified grounded theory approach, based on Strauss and Corbin's version. The inclusion of existing experiences and knowledge, especially in data analysis and theory generation during systematic comparison, was a consideration in selecting the research methodology (Strauss and Corbin, 1998). Moreover, researchers often build new knowledge on existing knowledge for cumulative theory development (Goldkuhl and Cronholm, 2010). For that, ignoring existing knowledge tends to be at the risk of reinventing the wheel.

Furthermore, in the context of this study, the existing knowledge is also used for minding the scattering amplitude of the collected data to be reasonably confined and manageable. In order to establish the study within a reasonable confinement, it refers to the perspectives of Facione and Schoenfeld for critical thinking and mathematical thinking, respectively. Strauss & Corbin (1990, 1998) asserted that grounded theory is an action/interactional method of theory building and allows analytic tools or techniques to be used during data analysis process. Either by using a variety of techniques, matrices or computers, researchers need ways of probing into and organizing data (Strauss and Corbin, 1998). In accordance with their assertion, this study uses research tools as data-oriented conceptual clarification to support grounded theory analysis and interpretation by linking categories more clearly to the data, namely, the conditional relationship guide and the reflective coding matrix. The conditional relationship guide contextualizes the central phenomenon and relates categories linking structure with process (Scott and Howell, 2008; Scott, 2004). This paper provides detailed explanation of the usage of conditional relationship guide in the axial coding process.

2.0 Methods

2.1 Data Acquisition

Data acquisition is oriented to grounded theory approach, which involves multiple stages of data generation and collection. Data were generated from semi-structured interviews with eight participants. Participants of the study comprised experts from two civil engineering consultancy firms in southern region of West Malaysia, preferably those who have been involved in this profession for at least five years. The time duration for each interview was about two hours. The interviews were audio-recorded and transcribed by the researcher.

Additionally, data were collected from the pertinent literatures and documents. Constant comparative method for analyzing data in grounded theory treated literature as 'data' and repetitively compared it with the emerging categories to be well integrated in the theory. The properties and dimensions brought out from the comparison method against the literature, were used to examine the incident in the data (Strauss and Corbin, 1998). This study used two types of sampling methods, namely purposive sampling and theoretical sampling.

2.2 Ethical Considerations

The researcher adhered to some of the techniques suggested by Johnson and Christensen (2000). For this purpose, a consent letter was prepared and given to each participant prior to the interview session. This informed consent states the objective of conducting the research and the assurance of the anonymity and the confidentiality of the participants. Participants were also assured that no intention to inflict any harm and their participations are voluntary and they may stop at any time without repercussion if feel uncomfortable. It is essential to make the participants understand that the research and their participations are important for the authentic and reliable data sources.

3.0 Data Analysis

The data acquisition and analysis of this study are interrelated process (Corbin and Strauss, 1990). This is to allow the occurrence of two analytic procedures pertaining to the constant comparative method of analysis and the asking of questions (Strauss and Corbin, 1990, 1998). Both are basic to the coding process and the typical procedure in grounded theory. There are three basic analytic processes in grounded theory namely open coding, axial coding and selective coding (Strauss and Corbin, 1990, 1998). This paper concentrates on explaining the open coding and axial coding process only. The selection of pertinent elements of critical thinking and mathematical thinking was executed at the beginning of the analysis. Open coding explains the selection process of the pertinent elements of critical thinking and mathematical thinking. Axial coding seeks for the interrelation among the pertinent elements in the real-world civil engineering practice. Data were analyzed solely by the researcher. Coding process was done manually. However, the analysis and emergent codes and categories were reviewed and verified by the experts in those particular fields to ensure trustworthiness. Microsoft Words 2010 and Microsoft Excel 2010 were used to assist the organization and management of data.

3.1 Open Coding – Selection Process of Pertinent Elements

Open coding is a process of breaking down, examining, comparing, conceptualizing, and categorizing data (Strauss and Corbin, 1990). The comparative method engages the procedures of asking questions and making comparisons was being used in this open coding process. By using the hybrid approach of grounded theory analysis, inductive and deductive approaches were integrated during the open coding process. Inductive codes were generated by directly examining and interpreting the data, which were embedded in the transcripts of interviews. In this study, the open coding was done mostly by line to line coding. The codes were named either by using in-vivo codes or in-vitro codes, as some of them were named after constructs already existing in other theories, if these names seemed to fit best, and when creating new ones would not be practical or justified (Enko, 2014). The selection of pertinent elements of critical thinking and mathematical thinking was done during the open coding process. Examples of open coding are shown in the Table 1.

Table 1: Examples of Open Coding

Transcript	*Open Codes*
We will check the platform level, will ask earthwork department, which platform is high and low, for structure, we look at the layout for the pipelines. We do deal with other departments such as infra for water sewage and drain, structure and earthwork	Comprehending ideas Communicating Examining ideas Team working
We have ISO documents, work flowchart, so, we follow…	ISO document Work flowchart

The researcher audio-recorded and transcribed all interviews. The transcripts of the interview were the main data source of this study. Each transcript was coded inductively. The open coding process was initiated on the first transcript as soon as it was transcribed closely after the first interview. As data acquisition and analysis were run concurrently, each interview led to further subsequent interviews as new information and categories emerged from previous interview data analysis (Johnson and Christensen, 2000). The emergent categories derived from data determined the orientation of the following interview. It is known as doing theoretical sampling. The theoretical sampling was employed from the first interview or data collection (Birks and Mills, 2011).

Through the theoretical sampling, the appropriate and relevant interview questions and interviewees were determined based on the concepts and categories generated from the data. Nevertheless, the core idea embedded in the interview protocol which related to the research questions was maintained to minimize variation among data generation. Constant comparative process initiated with the first interview. Comparison was made between data and data, coding and data, coding and coding, with the previous analysed transcripts helped a lot the open coding process. It ensured the same meaning of interpretation, differentiating codes for the same data segment (multiple codes) or simultaneous codes (applies two or more codes within a single datum), keeping track and avoiding ambiguous guess (Saldaña, 2009). The iterative process of interviewing-coding-comparing-interviewing was continuously carried out until reach the saturation level in which no more new theme and concept are emerged from the new data acquisition.

Initially, the inductive codes obtained were classified as critical thinking or mathematical thinking, through the lens of Facione's critical thinking skills and dispositions and aspects of Schoenfeld's mathematical thinking. The core skills of critical thinking were abbreviated: interpretation (CIP), analysis (CAN), evaluation (CEV), inference (CIF), explanation (CEX) and self-reflection (CSR). The same for dispositions of critical thinking: truth-seeking (CDT), open-mindedness (CDM), analyticity (CDA), orderliness (CDO), confidence (CDC), inquisitiveness (CDI), and maturity of judgment (CDR). Whereas, five cognitive aspects of Schoenfeld's mathematical thinking are knowledge base (MKB), problem solving strategies (MPS), monitoring and control (MMC), beliefs and affects (MBA), and practices (MMP).

Next, the total repetition number of open codes for core skills of critical thinking was determined. Then, it was followed by tabulating the open codes with repetition number for each skill. The same procedure was applied to the dispositions of critical thinking and aspects of mathematical thinking. After that, the total repetition number of open codes of critical thinking and mathematical thinking was determined. All the emergent open codes were categorized into two, either as major open code or category. Major open code was open code that represents a collective meaning of the code from the participants. Category was an abstraction of few related open codes. Subsequently, all the open codes were listed down to identify pertinent elements of critical thinking and mathematical thinking. The pertinent elements consisted of selected major open codes or categories, were identified according to their predominant pattern and frequency in the listing.

Then, the selected open codes and categories were eventually discussed with experts for verification. The pertinent elements and their related core skills of critical thinking are shown in Appendix 1. Appendix 2 shows the pertinent elements and their related dispositions of critical thinking. The pertinent elements and their related cognitive aspects of mathematical thinking are shown in Appendix 3. As a summarization, Table 2 shows the total number of pertinent elements of critical thinking and mathematical thinking. These groupings were treated as the main reference for the next stage of data analysis. Subsequently, the groupings were extended as the analysis progress that provides the foundation to the logic diagrams done during the axial coding (Strauss and Corbin, 1998). For this purpose, the research tool, namely conditional relationship guide was used during the axial coding process.

Table 2: Total number of pertinent elements of critical thinking and mathematical thinking

CTS, CTD and MTC	Critical Thinking													Mathematical Thinking				
	Core Skills (CTS)						Dispositions (CTD)							Cognitive Aspects (MTC)				
	CIP	CAN	CEV	CIF	CEX	CSR	CDT	CDM	CDA	CDO	CDC	CDI	CDR	MKB	MPS	MMC	MBA	MMP
Number of pertinent elements	2	3	2	2	2	6	1	2	2	1	1	1	1	4	6	8	4	5
Total number of pertinent elements	**17**						**9**							**27**				

3.2 Axial Coding- Interrelation among Pertinent Elements

Axial coding is an intermediate stage of coding process where those deconstructed data during open coding are gathered back together in new form by creating associations between a category and its subcategories, in which, open coding and axial coding go hand in hand (Corbin and Strauss, 2008; Strauss and Corbin, 1990). In other words, axial coding consists of two ways of operation; firstly is to develop fully individual categories by connecting subcategories, completely developing the range of properties and their dimensions, and secondly is to link categories together (Birks and Mills, 2011).

The aim of axial coding process, together with the memos written during this process, is to relate and continue generating categories in terms of their properties and dimensions (Strauss and Corbin, 1998). Thus, when analysts code axially, they look for answers to questions such as why or how come, where, when, how, and with what results, and in so doing they uncover relationships among categories. This is to bringing the process, action/interaction of the area of study into analysis. In this study, the researcher used the analytic tools to relate and integrate categories in understanding the interaction among the pertinent elements. For this axial coding process, the researcher concentrated on relating and understanding the interrelation among the pertinent elements (major open codes or categories). The process was visualized through the conditional relationship guide. Codes (italicized) were used to define each category relatively, as shown in Table

3. The conditional relationship guide was utilized to clarify the process. It contextualizes the central phenomenon and relates categories structure with process, which specifically engages Strauss and Corbin's relational investigative questions (Scott and Howell, 2008).

Table 3: Example of Conditional Relationship Guide for Categories

Categories	**What**	**Where**	**When**	**Why**	**How**
Clarify meaning	At preliminary stage, after getting the architecture drawing, we have to study it and determine our layout structure	*Preliminary stage*	Interpretation	*Concern behaviour in making decision*	*Diligence in seeking info; Coming to grips with uncertainty; Making conjectures*
Comprehending	When we want to design, we must aware of all the changes, and to understand the meaning of the changes	*Design stage*	Interpretation	*Correcting Confirming*	*Gathering relevant info*

That analytic tool consists of six columns; category, what, where, when, why, how and consequence, and was formatted to ask and answer each relational questions about the category named in the left column. It helped the researcher to understand the interactions of the pertinent elements by asking why and how questions that giving ideas of dynamic process over time. How and why questions were raised and case descriptions made for describing the chosen core category and the central phenomenon that was identified during the earlier phases of coding (Tuomela, 2005). Although the study reports record in time, the participants continue to interact with their realities (Scott and Howell, 2008) and this dynamic element is known as process (Strauss and Corbin, 1998). These questions provided insights in leading the researcher to the participant's mode of understanding the consequences. These consequences were the key categories where all other categories were focused. As a result, the interrelation among the pertinent elements was identified. The consequences are the group that will be primarily focused on, during the selective coding stage.

Memos helped in developing linkage among categories. Usually, early written memos carry only little reflective information which may reflect uncertainty, misconceptions, and feeble attempts, but with time, the data will become clearer and that the content of memos will be better in depth and quality of conceptualization (Strauss and Corbin, 1998). Examples of relating categories by bringing the process into analysis, using the research tool, are shown in Table 3. The example is drawn from the axial coding process of this study.

The table was completed by selecting a category and placing the category named in far-left column. This process was applied to all the fifty three categories, or also known as pertinent elements, identified in the study. Each relational question about the category (Scott and Howell, 2008; Scott, 2004), which is discussed below, needs to be posed to have a clear understanding of a conceptualized phenomenon.

What is [the category]?: It is content determination. It was defined either by using collective definitions based on codes or using the words of participant(s) that seems to capture the collective meaning of the category. Mostly, in this study, the researcher prefers to use the words of participant to avoid bias. For example, for the category named ‘adapting new approach / experience’, the researcher used quotation from the participant to answer it: ‘*The longer he involves in a field, the more experience he gains, which can be adapted to the next projects’*

Where / When does [the category] occur? : In this context of study, for ‘Where’ question was answered using ‘in’: in the design stage, in the preliminary stage. Whereas for ‘When’ question was answered using ‘during’: during analysing, during explaining, during making inference. To conclude, the researcher has chosen to be more specific in answering ‘when’ question and to answer more broadly in ‘where’ question. For instance, where and when do the ‘adapting new approach / experience’ occur? The answers were *‘in the stage of design and construction’, during ‘monitoring and control’.*

Why does [the category] occur? : To answer this question, the researcher chose the related pertinent elements for the particular category. The selected major open codes or categories, those giving overarching meaning of purposes, are chosen for answering this question. For instance, why does ‘adapting new approach / experience’ occur? The answer was *because of giving alternative ways and for staying well-informed*.

How does [the category] occur? : This question is showing action/interaction among the pertinent elements of that particular category. It brings the idea of dynamic process over time into the analysis. It gives great influence in determining the participant’s mode of understanding the consequences. For this, the researcher chose the related pertinent elements for the particular category, those offering more to the meaning of processes. For instance, how does ‘adapting new approach / experience’ occur? *By justifying, conforming, and having discussion.*

With what consequence does [the category] occur or is [the category] understood? : The consequence is what the participants get purposely and intentionally, such as experience and living. For this example of category, ‘adapting new approach /

experience', its consequences were *self-regulation, selecting/pursuing the right approach, and how efficient knowledge / experience is used.*

4.0 Discussion

A total of fifty three selected categories emerged from about two hundreds open codes during the open coding process. These selected categories were pertinent elements of critical thinking and mathematical thinking, which were mainly used in the real-world practice. The selection of these pertinent elements was based on the predominant pattern and frequency of the informants and open codes.

Each of the fifty three categories was explained in depth in axial coding process. Thus, the interrelation between categories was developed by answering the questions, what, where, when, why, how and with what consequences. The interrelation among pertinent elements developed during the axial coding process, together with the information interpreted from Table 2 has shown some interaction among the pertinent elements. It showed that total numbers of pertinent elements of critical thinking for both critical thinking skills and dispositions, and mathematical thinking were showing about the same figure. It suggested that in the design process:

- Both critical thinking and mathematical thinking were interwoven
- Both critical thinking and mathematical thinking were concurrently used
- Both critical thinking and mathematical thinking were indispensable
- Both critical thinking and mathematical thinking were inexorably linked

Those suggestions were supported by the excerpts below:

> *In designing, CT and MT are surely combined because both are concurrently used. So, it is good if can be applied to students, this understanding of CT and MT, because maybe basic knowledge can be given, like how to tackle a problem of a case, including communication……CT and MT run concurrently, we use both thinking in designing… (Engineer 3)*

> *When we cannot get the answer from software, we have to manipulate the equation for getting another calculation. For obtaining that formula or the way to solve the problem, we have to apply a method or a skill, and CT is the thing that we have to have. It means, we use CT to think of how to manipulate the mathematical formula itself. (Engineer 3)*

> *Sometimes what we do, not saying it is wrong, but, construction-wise, it is difficult to be done. So, we have to think of other alternative to be done.*

Meaning, we use CT to think of other alternative or to set tolerance to our design. (Engineer 3)

CT is predominantly used at the early stage of designing, and then, we use all the sources we have to smoothen our design work….both CT and MT are used, cannot stop just like that, they are indeed combined together…(Engineer 4)

5.0 Conclusion

Inspired by the evolvement of grounded theory and the appropriateness of answering the research questions in a reasonable confinement, modified grounded theory was chosen as a considerable approach in this study. This paper describes parts of the coding process in grounded theory. It aims to answer the research questions on what are the pertinent elements of critical thinking and mathematical thinking used in the real-world civil engineering practice and how do the pertinent elements interrelate among each other during the execution of the design practice. The questions were answered by the identification of the fifty three pertinent elements of critical thinking and mathematical thinking during the open coding process. Pertinent elements were identified from the emergent codes during the open coding process, with reference to the Facione's core skills and dispositions of critical thinking and five cognitive aspects of Schoenfeld's mathematical thinking. Subsequently, the interrelation among the pertinent elements was empirically developed through the axial coding process, using the conditional relationship guide. The findings serve a basis for the development of the selective coding. It provides the main source of information to explain the interaction among pertinent elements in the selective coding process. The study contributes useful information to engineering education instruction, which is aligned with the expectations of engineering program outcomes set by the Engineering Accreditation Council.

6.0 Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Reference

Beyer, B. K. (1990). What Philosophy Offers to the Teaching of Thinking. *Educational Leadership*, 55–60.

Birks, M., & Mills, J. (2011). *Grounded Theory: A Practical Guide*. *Grounded Theory: A Practical Guide*. Los Angeles, CA: Sage Publications Ltd.

Corbin, J., & Strauss, A. (1990). Grounded Theory Research_procedures, canons and evaluative criteria. *Qualitative Sociology*, *19*(6), 418–427.

Corbin, J., & Strauss, A. (2008). *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory* (3rd ed.). Thousand Oaks, CA: SAGE.

Devlin, K. J. (2002). The Math Gene : How Mathematical Thinking Are Like Gossip Where Mathematics Comes From : How the Embodied Mind Brings Mathematics Into Beino, 74–78.

Devlin, K. J. (2012). *Introduction to Mathematical Thinking*. Theoklesia, LLC.

Enko, J. (2014). Creative writers' experience of self-determination: An examination within the grounded theory framework. *Thinking Skills and Creativity*, *14*, 1–10. doi:10.1016/j.tsc.2014.06.004

Facione, P. A. (1990). Critical Thinking : A Statement of Expert Consensus For Purposes of Educational Assessment and Instruction. *California Academic Press*.

Facione, P. A. (2007). Critical Thinking : What It Is and Why It Counts. *California Academic Press*, 1–23.

Facione, P. A. (2013). Critical Thinking : What It Is and Why It Counts. California Academic Press.

Facione, P. A., Facione, N. C., & Giancarlo, C. A. (2000). The Disposition Toward Critical Thinking : Its Character , Measurement , and Relationship to Critical Thinking Skill. *Informal Logic*, *20*(1), 61–84.

Goldkuhl, G., & Cronholm, S. (2010). Adding Theoretical Grounding to Grounded Theory : Toward Multi-Grounded Theory. *International Journal of Qualitative Methods*, *9*(2), 187–205.

Johnson, B., & Christensen, L. (2000). *Educational Research: Quantitative and qualitative approaches*. Needham Heights, MA: Allyn & Bacon.

Kadir, M. A. A. (2007). Critical thinking: A family resemblance in conceptions. *Education and Human Development*, *1*(2), 1–11.

Katagiri, S. (2004). *Mathematical Thinking and How to Teach It*. Tokyo: Meijitosyo Publishers.

Paul, R. (1995). *Critical Thinking: How to Prepare Students for a Rapidly Changing World*. Santa Rosa, CA: Foundation for Critical Thinking.

Radzi, N. M., Abu, M. S., Mohammad, S., & Abdullah, F. A. P. (2011). Math-oriented critical thinking elements for civil engineering undergraduates : are they relevant ? In *IETEC'11 Conference*. Kuala Lumpur, Malaysia.

Radzi, N. M., Mohamad, S., Abu, M. S., & Phang, F. A. (2012). Are Math-Oriented Critical Thinking Elements in Civil Engineering Workplace Problems Significant?: Insights from Preliminary Data and Analysis. *Procedia - Social and Behavioral Sciences*, *56*, 96–107.

Saldaña, J. (2009). *The Coding Manual for Qualitative Researchers*. London: Sage Publications Ltd.

Schoenfeld, A. H. (1985). *Mathematical Problem Solving*. Orlando: Academic Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: MacMillan.

Scott, K. W. (2004). Relating Categories in Grounded Theory Analysis : Using a Conditional Relationship Guide and Reflective Coding Matrix. *The Qualitative Report*, *9*(1), 113–126.

Scott, K. W., & Howell, D. (2008). Clarifying Analysis and Interpretation in Grounded Theory : Using a Conditional Relationship Guide and Reflective Coding Matrix. *International Journal of Qualitative Methods*, *7*(2), 1–15.

Stacey, K. (2007). *What is Mathematical Thinking and why is it important? Progress report of the APEC- project Collaborative Studies on Innovations for Teaching and Learning Mathematics in Different Cultures (II) Lesson Study focused on Mathematical Thinking.*

Sternberg, R. J. (2012). What Is Mathematical Thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The Nature of Mathematical Thinking* (pp. 303–316). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Strauss, A., & Corbin, J. (1990). *Basics of Qualitative Research: Grounded theory procedures and techniques*. Newbury Park, California: Sage Publications, Inc.

Strauss, A., & Corbin, J. (1998). *Basics of Qualitative Research: Techniques and Procedures for Developing Grounded Theory* (2nd ed.). California: Sage Publications, Inc.

Tuomela, A. (2005). *Network Service Organisation - Interaction In Workplace Networks*. Helsinki University of Technology.

Appendices

Appendix 1: Open Coding – Pertinent Elements and Related Core Skills of Critical Thinking

Pertinent Elements (Major Open Codes / Categories)	Core Skills of Critical Thinking
Comprehending	Interpretation (CIP)
Clarifying meaning	
Examining Ideas / output	Analysis (CAN)
Checking thoroughly	
Detecting failure	
Assessing credibility of output / info	Evaluation (CEV)
Revising / Reanalyse design	
Considering relevant info	Inference (CIF)
Drawing reasonable conclusion	
Justifying reasonably	Explanation (CEX)
Defending with good reasons	
Counter checking	Self-reflection (CSR)
Correcting / Self correction	
Confirming	
Self-consciously to thinking / self-consciousness	
Complying	
Amending	

Appendix 1: Open Coding – Pertinent Elements and Related Dispositions of Critical Thinking

Pertinent Elements (Major Open Codes / Categories)	Dispositions of Critical Thinking
Flexibility in considering alternatives	Truth seeking (CDT)
Understanding others' opinions	Open Mindedness (CDM)
Tolerant of divergent views	
Anticipating the results	Analyticity (CDA)
Using evident to solve problems	
Diligence in seeking info	Orderliness (CDO)
Confidence in reasoning	Confidence (CDC)
Intellectual curiosity	Inquisitiveness (CDI)
Careful and prudent	Maturity (CDR)

Appendix 2: Open Coding – Pertinent Elements and Related Aspects of Cognition of Mathematical Thinking

Pertinent Elements (Major Open Codes / Categories)	Cognitive Aspects of Mathematical Thinking
Informal knowledge / Intuition / imagining	Cognitive mathematical knowledge base (MKB)
Engineering sense	
Applying / transferring maths knowledge / theory	
Using standard equation/formula/algorithm	
Looking for patterns	Problem solving strategies / heuristics (MPS)
Working backward	
Analytical reasoning skills	
Simulate real life experience	
Solving open-ended questions	
Gathering info/data/relevant info	
Selecting / Pursuing the right approach	Monitoring and control (MMC)
Concern behaviour in making decision	
Having discussion	
Self-regulation	
Decision to be made along the way	
Conforming	
How efficient knowledge / experience is used	
Adapting new/different approach/situation/experience	
Dominating orientation	Belief and affects (MBA)
Giving alternative ways / solutions	
Maths consciousness/ consciousness in assessing material	
Mathematical proficiency	
Having mathematical views and sense-making	Mathematical practices (MMP)
Forming conjectures / assumption	
Manipulating formula / input data/symbols/ equation	
Defending claims mathematically	
Coming to grip with uncertainties	