**MATHEMATICAL PSYCHIATRY OF FIELD PLATE LOAD TEST USING FINITE ELEMENT METHOD**

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**Abstract:** Present investigation deals with a mathematical psychiatry of Field Plate Load Test (FPLT) using FEM (Finite Element Method) in clayey soil to evolve the effect of plate dimension on settlement and coefficient of subgrade reaction ($k_s$). A comparison between the obtained FEM results using Mohr-Coulomb’s soil model and interpretation of prior review of literature was addressed. Outcomes revealed that the $k_s$ is strictly dependent on parameters like size of the loaded area and loading magnitude, and thus completely general and generic, and not a fundamental material property of soil that can somehow be determined rationally.

**Keywords:** Field plate load test (FPLT), Winkler’s model, finite element method (FEM), coefficient of subgrade reaction ($k_s$)

1.0 Introduction

Field Plate Load Test (FPLT) is one of the important tests especially in obtaining the necessary information about the soil with particular reference to design of foundation which has certain limitations (Bowless, 1996). Test results can imitate the character of soil situated within a depth of less than twice the width of the bearing plate. In general, since the foundations are larger than the test plates, settlements and shear resistance will depends on the properties of a much thicker layer. Furthermore, it does not give the ultimate settlements particularly in case of cohesive soils. Thus the results of the test are likely to be misleading at shallow depths. The test is trustworthy only if the sand is reasonably identical over a significant depth of the complete foundation. Ideally, field plate bearing tests are to be carried out at different depths with varying sizes in order to make the extrapolations, but this is generally ruled out on economic grounds and further problems would be introduced if the tests had to be carried out below the level of ground water table. There is momentous spread out in comparing the relationship between plate size and settlement for a given applied stress. The former field studies have indicated that the limit of plate settlement to the applied stress of range between 3 and 5 rather than holding a fixed value of 4 (Bjerrum and Eggestad, 1963).
In the recent, numerous investigators extended classical mathematical (Winkler’s model), where the behaviour of soil is simplified by means of fictitious springs placed continuously underneath the structure denoted as the coefficient of subgrade reaction (k_s) or coefficient of elastic uniform compression of the soil (C_u). These spring constants are bringing relationship between the applied load (pressure) and deflection (settlement) below a structural element founded on elastic half space which mainly relies on its void ratio, stress history and loading rate. In granular soils, it is a function of depth of strata, while in cohesive soils distinctly influenced by the moisture content. The k_s is not an intrinsic soil property but it is a response to a given load over a given area and depends not only on the deformation characteristics of the soil but also on the size of contact area between plate and subgrade. Due to its great sensitivity in sampling disturbance, accurate evaluation of k_s in the laboratory is extremely difficult. It depends on many factors, such as shape of the foundation, stiffness of foundation slab, shape of loading on the foundation, depth of the loaded area below the ground surface, and time (Iancu and Ionut, 2009).

Using Winkler’s hypothesis, former computer codes have been developed by numerous researchers to analyze the positioned technical parameters of elastic half space. If the soil having varied stratum with dissimilar thickness, the value of equivalent k_s has to be at least a function of the thickness of soil layer. Terzaghi (1955) made recommendations on k_s for rigid slab positioned on a soil medium, but it was not specific to larger slab. Biot (1937) solved the problem for infinite beam with a concentrated load resting on 3D elastic soil continuum. The correlation of continuum elastic theory was equated where the maximum moments are available.

The location of plate load test can be conducted based on exploratory borings at foundation level under the worst estimated conditions. In case, water table within the depth of equal to the width of the test plate can be conducted at water table level. If the water table is higher than the test level, it can be lowered to the test level and maintained by pumping through a sump, away from the test plate. The set-up of FPLT is presented in Figure 1.
Winkler’s model is well known for bringing out the relationships of pressure – settlement on elastic half space to eliminate the bearing soil reaction as a variable in the problem solution. In this hypothesis, the soil medium is a system of identical, independent, closely spaced, discrete and linearly elastic springs and the ratio between contact pressure, \( p \), and settlement, \( w \), produced by load application at an arbitrary point, \( i \), on the contact surface, is given by the coefficient of subgrade reaction, \( k_s \). The numerical expression presented in Eq. 1:

\[
k_s = \frac{\text{Pressure}}{\text{Settlement}}
\]

The significant shortcomings in evaluating \( k_s \) by no means an intrinsic property of the elastic half space. But great care is required for owing the problem dependent on the nature of the parameter (Terzaghi et al., 1966). The obtained relationship between applied pressures against settlement is presented in Figure 2. The elucidation of test results (deformation properties) is usually made using isotropic elastic theory because of its simplicity. Hence geotechnical engineering parameters such as Young’s modulus (\( E_s \)) and coefficient of subgrade reaction (\( k_s \)) can be expressed as follows.
Using a rigid surface of plate diameter, \( D \) with applied pressure, \( p \) on a semi-infinite, isotropic soil characterized by Young’s modulus, \( E_s \) and Poisson’s ratio, \( \mu \) is given by

\[
w_1 = \frac{\pi p_i D (1-\mu^2)}{4 E_s}
\]  

Young’s modulus \( (E_s) \) can be evaluated by

\[
E_s = \frac{\pi p_i}{4 w_1} D (1 - \mu^2)
\]  

The modulus of subgrade reaction, \( k_s \), can be calculated from the initial slope of the curve Figure 2 until the limit pressure, \( p_l \) is reached. The following equation, which is produced by the theory of elasticity solution (Timoshenko, 1951), in comparison of equation (Eqns. 1 and 2), used to determine the value of \( k_s \) as

\[
k_s = \frac{4E_s}{\pi D (1-\mu^2)}
\]
For clayey and silty soils and for loose to medium dense sandy soils with standard penetration resistance, \( N < 15 \), a 450 mm square plate or concrete blocks shall be used. In case of dense sandy or gravelly soils (\( 15 < N < 30 \)) at least of three plates varying between 300 to 750 mm shall be used depending upon practical considerations of reaction loading and maximum grain size. The side of the plate shall be at least four times the maximum size of the soil particles present at the test location (IS: 1888, 1997).

2.0 Methodology

In the present exploration, Finite Element Method (FEM) analyses were performed by controlling the settlement and boundary conditions applied to the soil surface below the loading plate. An axis-symmetry mesh described by Mohr – Coulomb failure model was chosen. The domain radius and heights are 5\( D \) as mentioned in the literature (Azizi, 2000; Desai and Christian, 1977; Potts and Zdravkovic, 1999). A 15-noded triangular element with fourth order interpolation for settlements of the numerical integration was used to define the finite element mesh as shown in Figure 3.

![Figure 3: Geometry and Mesh of FPLT in FEM](image-url)
The performance of ground mainly depends on current stresses and strains involved in the study. Therefore, it is essential to prescribe the stress conditions, such as in the first phase initial soil stresses are generated and loading continued. The soil behaviour described by the Mohr – Coulomb’s model, having the material properties such as Young’s modulus, \( E_s = 30 \text{ MPa} \), Poisson’s ratio, \( \mu = 0.35 \), cohesion, \( c = 69 \text{ kN/m}^2 \), dry unit weight of soil, \( \gamma_d = 18.8 \text{ kN/m}^3 \), saturated unit weight, \( \gamma_{sat} = 20 \text{ kN/m}^3 \) and the angle of internal friction or shearing resistance, \( \phi = 32^\circ \). The soil considered in the study is a compressible red soil and has cohesion. For this soil, the basic tests were conducted in the laboratory for its characterization. As per the basic properties of soil is concerned, it is in red color and has no gravel. The IS classification of soil is clayey sand (SC).

3.0 Results and Discussion

The results obtained from FPLT can be explicitly used to estimate the settlement of a footing and a little geotechnical parameters also can be derived. Amongst them, the Young’s modulus (\( E_s \)) and coefficient of subgrade reaction (\( k_s \)) are of most attention. The test results reflect only the characteristics of soil located within the depth of less than twice the width of bearing plate. Since the foundations are generally larger than the test plates, settlement and shear resistance will depend on the properties of a much thicker stratum.

Ten finite element analyses were performed to extend the relationship between applied pressure and settlement as shown in Figure 4 for the plate diameters ranging from 300 to 750 mm. Keeping the hypothesis in view, the plate settlement is the same of an elastic half space, until the limit pressure is reached and the young’s modulus, \( E_s \) can be expressed from results of the FPLT in terms of the ratio of bearing pressure to plate settlement, as presented in equation (Eq. 3). It is well known that, the settlement of plate is uniform of an elastic half space until it reaches to limit equilibrium. This integration is not truly justified because under the edges of applied pressure area may leads to a local punch failure and thus no more being an elastic equilibrium in all points underneath the plate. Thus, Boussinesq’s elucidation may lead to flawed outcomes especially in case of cohesion less soil with low punch strength.
In general, relative shear stress is defined as the ratio between maximum shear stress or radius of Mohr circle and the maximum value of shear stresses for the case where Mohr’s envelope is expanded to touch the Mohr-Coulomb failure envelop as presented in equation (Eq. 5). The relative shear is defined as

\[
\tau_{rel} = \frac{\tau}{\tau_{max}}
\]  

(5)

Where \(\tau\) is the maximum value of shear stress (i.e., the radius of the Mohr circle) and \(\tau_{max}\) is the maximum value of shear stress for the case where Mohr’s circle is expanded to touch the Coulomb failure envelope keeping the intermediate principal stress is constant.

The plastified zone by means of relative shear stresses for plate dimensions between 300 to 750 mm is presented in Figures 5 (a) and (b) respectively. The shear stresses for the plate diameter, \(D = 300\) mm corresponds to an applied pressure of 110 kPa with prescribed displacement of 1.3 mm are increasing along with the plate dimension. For clayey soils, bearing capacity (from shear consideration) of a larger foundation is almost the same as that for smaller test plate. Especially in dense sandy soils, the bearing capacity increases with the size of foundation. Thus tests with smaller size plate tend to
give conservative values in dense sandy soils. Therefore it is necessary to test with the plates of at least three sizes and bearing capacity results can be extrapolated for the size of the actual foundation (minimum dimensions in the case of rectangular footings).

![Shear stress diagrams](image1)

**Figure 5:** Relative shear shadings, $\tau_{\text{max}}$

![Relationship graph](image2)

**Figure 6:** Relationship of young’s modulus with p/w
The relationship between young’s modulus (E_s) with the ration of pressure – settlement (p/w) as illustrated in Figure 6. Applying equation (Eq. 3) for each applied pressure – settlement curves are presented in Figure 3, the values of E_s was developed. From the Figure 6, it can be clearly seen that, for plate dimension of 300 mm, the bearing capacity of soil under the applied pressure consumes its elastic strain more instantaneously than the plate dimension of 750 mm. This is due to the contact area of elastic half space below the plate.

![Figure 7: Variation of k_s with plate dimension](image)

Figure 7 shows the relationship between plate dimension and k_s. The coefficient of subgrade reaction can be derived from equation (Eq. 4) for plate dimensions from 300 to 750 mm. It is renowned that the k_s vary according to the size of the plate used in FPLT. Thus, it has no unique value and mainly depends on the size of applied area and it decreases with increasing plate dimensions.
Figure 8 presents the relationship between plate diameter and settlement under same load per unit area. It is observed from the above figure that the bearing capacity of cohesive soils decreases with the plate dimension. For large plate diameters, settlement increases proportionally with the size of the plate dimensions.

4.0 Conclusions

FPLT is not an option for which doesn’t have homogeneous strata of at least a depth equal to 1.5 to 2 times the width of model foundation. The obtained results from present investigation revealed that a mathematical relationship among modulus of elasticity and coefficient of subgrade modulus are decreased with the increase in plate dimension. Likewise the settlement also decreased with increased plate side dimension. Finally, the relation between plate diameter and settlement under same load per unit area is in good agreement with some observation presented in literature (Caquot and Kerisel, 1968).

References